Near Subsurface Imaging

2019 RMMC Summer School
Inverse Problems in Imaging

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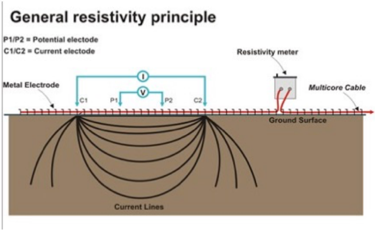
Near subsurface imaging

Boise Hydrogeophysical Research Site (BHRS)

- Field laboratory on a gravel bar adjacent to the Boise River, 15 km southeast of downtown Boise.
- Consists of coarse cobble and sand. Braided stream fluvial deposits overlie a clay layer at about 20 m depth.

Difference in retention properties in a lenticular sand feature yields significantly different geophysical properties.

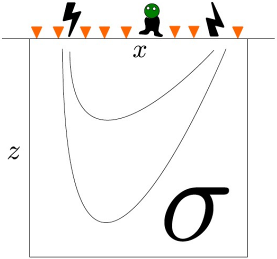
Electrical Resistivity Tomography (ERT)



- 2D grid of observations provides a 2.5-D inverted model that emphasizes the sand lenticular feature.
- BHRS survey consisted of 12 electrodes spaced 1 meter apart acquired with a dipole-dipole configuration.

BHRS survey acquired at surface when subsurface achieved saturation.

Electrical Resistivity Model



$$-\nabla \cdot \sigma \nabla \varphi = \mathbf{i}(\delta(x - s_+) - \delta(x - s_-))$$ [1]

φ - electric potential

$\mathbf{i}$ - current intensity

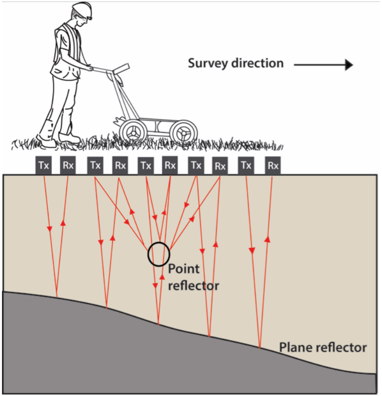
$s_\pm$ - source-sink position.

Model parameters: conductivity σ or resistivity $\rho = 1/\sigma$

Observed data: apparent resitivity $\frac{2\pi \triangle \varphi}{i} \kappa$

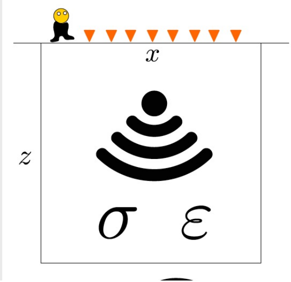
[1] Pidlisecky and Knight, 2008

Ground Penetrating Radar (GPR)



- GPR survey at BHRS acquired across center of gridded ER survey.
- GPR sampled line collinear with ER survey.
- GPR derived boundary gives constraint for inverting the ER dataset.

GPR Model



$$\begin{pmatrix} \mu_o & 0 & 0 \\ 0 & \mu_o & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \begin{pmatrix} \dot{H}_z \\ -\dot{H}_x \\ \dot{E}_y \end{pmatrix} = \begin{pmatrix} 0 & 0 & \partial_x \\ 0 & 0 & \partial_z \\ \partial_x & \partial_z & 0 \end{pmatrix} \begin{pmatrix} H_z \\ -H_x \\ E_y \end{pmatrix}$$

$$\sigma \begin{pmatrix} 0 \\ 0 \\ E_y \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -J_y \end{pmatrix},$$

E_y-electric field, (H_x, H_z) - magnetic field

μ_0-constant permeability, J_z-source

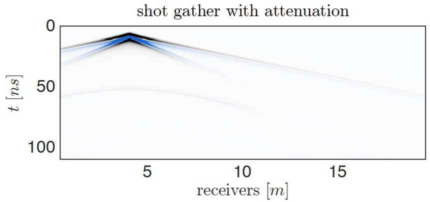
Model parameters: conductivity σ and permitivity ϵ

Observed data: electric current Mu

Complementary data in Subsurface Imaging

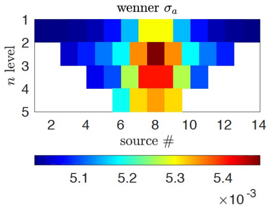
Ground Penetrating Radar

- High frequency
- Conductivity through attenuation and reflection



Electrical Resistivity

- Low frequency
- Directly sensitive to conductivity



GPR inversion

$$\mathbf{u} = \mathbf{L}_w \, \mathbf{s}_w ,$$

$$\mathbf{d}_w^s = \mathbf{M}_w \, \mathbf{u}$$

$$\min_{\boldsymbol{\varepsilon},\boldsymbol{\sigma}} \frac{1}{2} \left(E_{w,\varepsilon}(\boldsymbol{\varepsilon};\, \mathbf{d}_w^o) + E_{w,\sigma}(\boldsymbol{\sigma};\, \mathbf{d}_w^o) \right), \quad E_{w,\varepsilon} = \frac{1}{n_s} \sum_s E_{w,\varepsilon}^s, \quad E_{w,\varepsilon}^s = \frac{||\mathbf{e}_w||_2^2}{||\mathbf{d}_w^{o,s}||_2^2}$$

ER inversion

$$\mathbf{L}_{dc} \boldsymbol{\varphi} = \mathbf{s}_{dc},$$

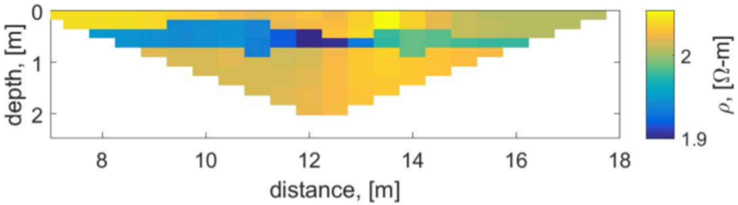
$$\mathbf{d}_{dc}^s = \mathbf{M}_{dc} \boldsymbol{\varphi},$$

$$\min_{\boldsymbol{\sigma}} E_{dc}(\boldsymbol{\sigma};\, \mathbf{d}_{dc}^o), \quad E_{dc} = \frac{1}{n_s} \sum_s E_{dc}^s, \quad E_{dc}^s = \frac{||\mathbf{e}_{dc}||_2^2}{||\mathbf{d}_{dc}^{o,s}||_2^2}$$

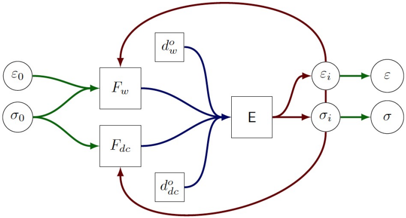
Boise Hydrogeophysical Research Site Results

- ER data inverted for resistivity
- Regularization in the form of subsurface boundary constraint inferred from GPR data

$\mathbf{u}$- electric field y component in space and time, $\mathbf{s}_w$ - source term, $\mathbf{M}_w$ - operator



Inverting ER and GPR jointly - full physics

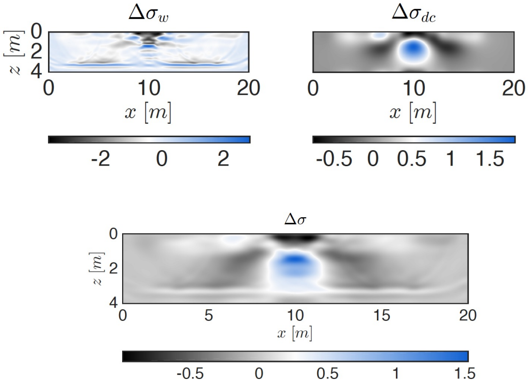


$$\boxed{\mathsf{E} = \mathsf{E}_w + \mathsf{E}_{dc}}$$

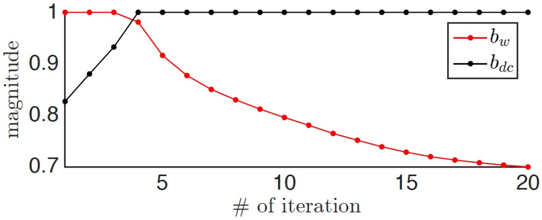
$$\varepsilon \leftarrow \varepsilon + \Delta\varepsilon$$

$$\sigma \leftarrow \sigma + \alpha\left(b_w\Delta\sigma_w + b_{dc}\Delta\sigma_{dc}\right)$$

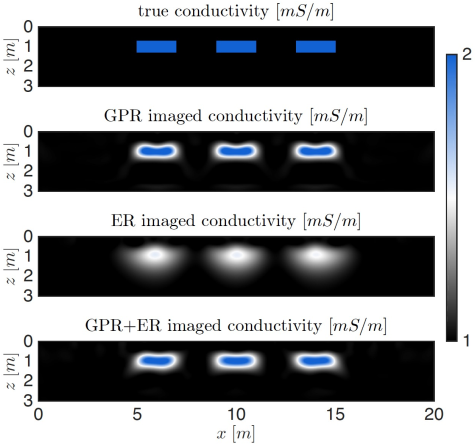
Combining updates



Data weights



Inverted images - full physics



Inverted cross section - full physics