Exercises

2019 RMMC Summer School
Inverse Problems in Imaging

Jodi Mead
Department of Mathematics

BOISE STATE UNIVERSITY

Exercise 1

a. Make sure you have IRTools software installed as described in *Nagy_Software_Instructions.pdf*. Run the setup script *WyomingInstall.m*

b. As described in *Nagy_L1d.pdf* generate the atmospheric blur test problem and plot the true solution:

```
[A, b_true, x_true, ProbInfo] = PRblurspeckle(32);
A = full(A);
PRshowb(x_true, ProbInfo);
```

c. Add noise to the data, estimate the solution with the *backslash* operator and plot it:

```
sigma=0.1;
b = PRnoise(b_true, sigma);
x_naive=A\b;
PRshowx(x_naive, ProbInfo);
```

Exercise 2

a. Go to https://math.boisestate.edu/~mead/WY_SS/ and download chi_obs.m and reg_matrix.m into the same directory you are running the MATLAB files in Exercise 1.

b. Continue with the estimates from Exercise 2. Generate an initial estimate of the parameters x0 and an estimate of the regularization parameter $\alpha = 1/\sigma_x$:

```
x0=zeros(size(x_true));
sigmax_guess=0.25;
```

c. Find a regularization parameter using the χ^2 test and extend the system of equations to include regularization:

```
[ A_reg,b_reg,my_alpha ] = reg_matrix(A,x0,b,sigma,sigmax_guess);
```

d. Find the regularized estimate with the *backslash* operator and plot it:

```
x_reg=A_reg\b_reg;
PRshowx(x_reg, ProbInfo);
```

Exercise 3

a. Go to and download resol_mat.m and conf_ellipsoid .m in the same directory you are doing the Exercises. Continue with the estimates from Exercise 2. Find the model and data resolution matrices and the posterior covariance matrix for the regularized parameter estimates:

   [Rm,Rd,covx] = resol_mat(A,my_alpha,b,sigma);

b. Plot the confidence ellipsoid for parameters x_4 and x_{10}:

   n1=4;n2=10;
   conf_ellipsoid(n1,n2,x_reg,covx)

c. What can you say about the statistical dependence of the parameters? Do you think the confidence ellipsoid accurately describes the uncertainty?

Exercise 4

a. Increase and decrease the data noise σ and note the corresponding change in the calculated $\alpha = 1/\sigma_x$ using the χ^2 test. What do you observe?

b. Plot the confidence ellipsoids for other pairs of parameters. Again make conclusions about the statistical dependence of the parameters and if the confidence ellipsoid accurately describes the uncertainty.