

Lecture Exercises  
Regression

1. Consider the multivariate data  $(x_1, x_2, y)$ :  $(-2, 4, 1), (-2, 1, 0), (0, 0, 2), (1, 1, 3)$  and assume we fit it to the line  $\hat{y}(x_1, x_2) = b_0 + b_1x_1 + b_2x_2$ .
  - (a) Identify the matrix  $\mathbf{X}$
  - (b) Identify the vector  $\mathbf{y}$
  - (c) Identify the formula for the coefficients  $\mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$ .
  - (d) The estimates of the coefficients are  $b_0 = 1.83, b_1 = 1, b_2 = 0.28$ . How would you interpret these values? For each change of 1 unit in  $x_1$ ,  $y$  changes 1 unit. If  $x_2$  is fixed, then for each change of 1 unit in  $x_1$ ,  $y$  changes 1 unit. For each change of 1 unit in  $x_1$ ,  $y$  changes 1.83 units. If  $x_2$  is fixed, then for each change of 1 unit in  $x_1$ ,  $y$  changes 1.83 units.
2. If  $\mathbf{X} = \begin{bmatrix} 1 & -2 & 4 \\ 1 & -2 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$  then the following values of  $\mathbf{b}$  and  $\mathbf{y}$  satisfy  $\mathbf{Xb} = \mathbf{y}$ .  $[1 \ 1 \ 1], [3 \ 0 \ 1 \ 3];$   
 $[1 \ 1 \ 1 \ 1], [4 \ -3 \ 6]; [1 \ 0 \ 1], [5 \ 2 \ 1 \ 2]; [1 \ 0 \ 0 \ 1], [2 \ -1 \ 5]; [0 \ 0 \ 1], [4 \ 1 \ 0 \ 1];$