Data Assimilation and Climate
AIMS Rwanda, March, 2020

Quiz 1
Introduction to Data Assimilation and Calculus of Variation

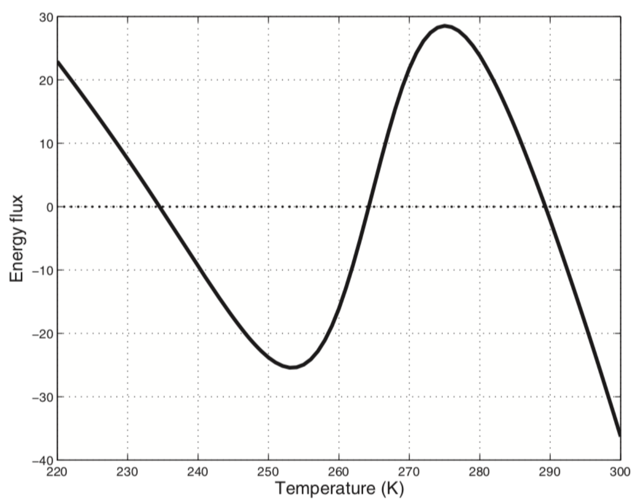
1. Consider the zero-dimensional energy balance model

$$C_p \frac{\partial T}{\partial t} = S(1-\alpha) - 4\epsilon\sigma T^4 \tag{1}$$

with $S$ solar energy, $\alpha$ reflectivity of Earth System, $\epsilon$ infrared transmissivity, $\sigma$ relates temperature to radiant emission, $C_p$ specific heat capacity of earth system and $T$ global mean surface temperature.

   (a) Use forward differences $\frac{\partial T}{\partial t} \approx \frac{T(t_{j+1})-T(t_j)}{dt}$ to approximate the derivate and assume that $dt = C_p$. Solve for $T(t_{j+1})$.

   (b) The following figure gives the right hand side of (1)




   If the initial temperature $T(t_1) = 240K$, estimate $T(t_2)$.

2. Consider a physical process in which real-valued state vables $x_i$, $i = 1, 2, ...,$ are generated according to the rule $x_{i+1} = f(x_i)$, where $f$ is a given real-valued function. The states are observed according to the data model $y_i = x_i + \epsilon_i$ , where the $\epsilon_i$ are independent random variables.

   Assume that observations $y_1, y_2, ..., y_N$ are available and that you want to estimate $x_1$. Describe in general terms how a cost function should be set up whose minimum is expected to give an estimate for $x_1$.