Data Assimilation and Climate
AIMS Rwanda, March, 2020

Quiz 2
Discrete Time Stochastic Process

1. Consider the general process model

$$\mathbf{X}(i) = \mathbf{M}_i \mathbf{X}(i-1) + \mathbf{b}(i) + \boldsymbol{\delta}_i, \quad i = 1, \ldots, N,$$

with $\mathbf{X}(0) \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ and where $\mathbf{X}(i), \mathbf{b}(i), \boldsymbol{\delta}(i) \in \mathbb{R}^n$ and $\mathbf{M}_i \in \mathbb{R}^{n \times n}$. Define $\overline{\mathbf{X}}(i)$ by

$$\overline{\mathbf{X}}(0) = \mathbf{0}; \quad \overline{\mathbf{X}}(i) = \mathbf{M}_i \overline{\mathbf{X}}(i-1) + \mathbf{b}(i), \quad i = 1, \ldots, N,$$

and set $\tilde{\mathbf{X}}(i) = \mathbf{X}(i) - \overline{\mathbf{X}}(i)$. Show that the $\tilde{\mathbf{X}}(i)$ satisfy the linear process model

$$\tilde{\mathbf{X}}(i) \quad = \quad \mathbf{M}_i \tilde{\mathbf{X}}(i-1) + \boldsymbol{\delta}_i, \quad i = 1, \ldots, N$$

with $\tilde{\mathbf{X}}(0) \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.