Data Assimilation and Climate
AIMS Rwanda, March, 2020

Lecture Activities
Climate Models

Consider the zero-dimensional energy balance model

$$C_p \frac{\partial T}{\partial t} = S(1-\alpha) - 4\epsilon\sigma T^4$$

with s solar energy, α reflectivity of Earth System, ϵ infrared transmissivity, σ relates temperature to radiant emission, C_p specific heat capacity of earth system and T global mean surface temperature.

1. Assume you approximate the first derivative with the forward difference $\frac{\partial T}{\partial t} \approx \frac{T(t_{j+1})-T(t_j)}{dt}$ where $T(t_j)$ is the global mean surface temperature at time t_j, $j = 0, \ldots$. Solve for $T(t_{j+1})$.

   Solution: $T(t_{j+1}) = T(t_j) + \frac{dt}{C_p} S(1-\alpha) - \frac{dt}{C_p} 4\epsilon\sigma T(t_j)^4$

2. Assume you approximate the first derivative with backward differences $\frac{\partial T}{\partial t} \approx \frac{T(t_j)-T(t_{j-1})}{dt}$. Solve for $T(t_j)$ and discuss how this approximation is different than the approximation in 1.

   Solution: $T(t_j) = T(t_{j-1}) + \frac{dt}{C_p} S(1-\alpha) - \frac{dt}{C_p} 4\epsilon\sigma T(t_j)^4$. Both approximations can find the temperate at different values of t_j. However, this approximation is implicit and would be more difficult to solve than the equation in 1.

3. What information do you need in order to estimate the global mean surface temperature?

   Solution: $S, \alpha, \epsilon, \sigma, C_p, dt, t_{end}, T_1$ (or T_0).

Data Assimilation: Find Initial Conditions T_1 (or T_0)