Data Assimilation and Climate
AIMS Rwanda, March, 2020

Make-up quiz

Consider that we want to estimate the state $\mathbf{x} \in \mathbb{R}^n$ given data $\mathbf{y} \in \mathbb{R}^m$ where

$$\begin{aligned} \mathbf{x} &= \mathbf{x}^b + \boldsymbol{\eta} \\ \mathbf{y} &= \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon} \end{aligned}$$

with $\mathbf{x}^b$ a background estimate for $\mathbf{x}$, and $\mathbf{H} \in \mathbb{R}^{m \times n}$ a measurement operator.

1. Find an estimate of the state that minimizes the errors in the background state ($\boldsymbol{\eta}$) and data ($\boldsymbol{\epsilon}$) in a weighted least squares sense, with weights $\mathbf{B}$ and $\mathbf{R}$.

2. Find an estimate of the state that maximizes the probability the data were observed. This is, find an estimate that maximizes the probability density function (pdf) of the errors in the state and data, where $\boldsymbol{\eta} \sim \mathcal{N}(0, \mathbf{B})$ and $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \mathbf{R})$